2.12 Finite Laguerre Series

A. Purpose

This subroutine computes the value of a finite sum of Laguerre polynomials,

$$y = \sum_{j=0}^{N} a_j L_j(x)$$

for a specified summation limit, N, argument, x, and sequence of coefficients, a_j. The Laguerre polynomials are defined in [1].

B. Usage

B.1 Program Prototype, Single Precision

INTEGER N

REAL X, Y, A(0:$m \geq$ N)

Assign values to X, N, and A(0), A(1), ..., A(N).

CALL SLASUM (X, N, A, Y)

The sum will be stored in Y.

B.2 Argument Definitions

X [in] Argument of the polynomials.

N [in] Highest degree of polynomials in sum.

A() [in] The coefficients must be given in A(J), J = 0, ..., N.

Y [out] Computed value of the sum.

B.3 Modifications for Double Precision

For double precision usage, change the REAL statement to DOUBLE PRECISION and change the subroutine name from SLASUM to DLASUM.

C. Examples and Remarks

See DRSLASUM and ODSLASUM for an example of the usage of SLASUM. DRSLASUM evaluates the following identity, the coefficients of which were obtained from Table 22.10, page 799, of [1].

$$z = y - w = 0,$$

where

$$y = 7.2L_0(x) - 3.2L_1(x) + 108L_2(x) - 144L_3(x) + 108L_4(x) - 43.2L_5(x) + 7.2L_6(x),$$

and

$$w = 0.01x^6.$$

D. Functional Description

The sum is evaluated by the following algorithm:

$$b_{N+2} = 0, \quad b_{N+1} = 0,$$
$$b_k = \frac{2k+1-x}{k+1} b_{k+1} - \frac{k+1}{k+2} b_{k+2} + a_k, \quad k = \mathrm{N}, ..., 0,$$
$$y = b_0.$$

For an error analysis applying to this algorithm see [2] and [3]. The first four Laguerre polynomials are

$$L_0(x) = 1, \quad L_1(x) = 1 - x,$$
$$L_2(x) = 1 - 2x + 0.5x^2,$$
$$L_3(x) = 1 - 3x + 1.5x^2 - (1/6)x^3.$$

For $k \geq 2$ the Laguerre polynomials satisfy the recurrence

$$kL_k(x) = (2k - 1 - x)L_{k-1}(x) - (k - 1)L_{k-2}(x).$$

The Laguerre polynomials are orthogonal relative to integration with the weight function e^{-x} over the interval $[0, \infty)$, thus

$$\int_0^\infty e^{-x} L_i(x) L_j(x)\, dx = 0 \quad \text{if } i \neq j.$$

Laguerre polynomials are normally used only with an argument x satisfying $x \geq 0$.

References

1. Milton Abramowitz and Irene A. Stegun, **Handbook of Mathematical Functions**, *Applied Mathematics Series 55*, National Bureau of Standards (1966) Chapter 22, 771–802.

2. E. W. Ng, *Direct summation of series involving higher transcendental functions*, **J. Comp. Phys. 3**, 2 (Oct. 1968) 334–338.

3. E. W. Ng, *Recursive algorithm for the computation of hypergeometric series*, **SIAM J. on Math. Anal. 2** (1971) 31–36.

E. Error procedures and Restrictions

The subroutine will return Y = 0 if N < 0. It is recommended that X satisfy X ≥ 0.

F. Supporting Information

The source language is ANSI Fortran 77.

Entry	Required Files
DLASUM	DLASUM
SLASUM	SLASUM

Based on a 1974 program by E.W. Ng, JPL. Present version by C.L. Lawson and S. Y. Chiu, JPL, 1983.

DRSLASUM

```
c      DRSLASUM
c>> 1994-10-19 DRSLASUM Krogh  Changes to use M77CON
c>> 1994-07-14 DRSLASUM CLL
c>> 1992-05-07 CLL
c>> 1992-04-28 DRSLASUM Replaced '1' in format.
c>> 1987-12-09 DRSLASUM Lawson  Initial Code.
c--S replaces "?": DR?LASUM, ?LASUM
c     Demonstration program for evaluation of a Laguerre series.
c     ------------------------------------------------------------------
      integer j
      real            x,a(0:6),y,w,dif,reldif,pn(6)
      data a / 7.2e0, -43.2e0, 108.0e0, -144.0e0,
     *         108.0e0, -43.2e0, 7.2e0 /
      data pn / 0.1e0, 0.3e0, 1.0e0, 3.0e0, 10.0e0, 30.0e0 /
c     ------------------------------------------------------------------
      print '(1x,3x,a1,12x,a1,14x,a3,9x,a6/)','x','y','dif','reldif'
      do 10 j = 1,6
        x = pn(j)
        call slasum (x, 6, a, y)
        w = 0.01e0 * (x**6)
        dif = y - w
        reldif = dif / w
        print '(1x,f6.2,3x,g15.8,2(3x,g10.3))',x,y,dif,reldif
   10 continue
      end
```

ODSLASUM

```
    x              y               dif          reldif

  0.10    -0.11444092E-04    -0.115E-04    -0.115E+04
  0.30     0.95367432E-05     0.225E-05     0.308
  1.00     0.10004997E-01     0.500E-05     0.500E-03
  3.00      7.2899961        -0.381E-05    -0.523E-06
 10.00      10000.000          0.00          0.00
 30.00      7290000.0          0.00          0.00
```