Constants and Functions of Mathematics: A Selection of Numerical Values[††]

$\pi \approx 3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282 3\ldots^{(\clubsuit)}$

$e \approx 2.71828182845904523536028747135266249775724709369995957496696762772407663035354759457138217852516642742746639193200 31\ldots^{(\clubsuit)}$

$\gamma \approx 0.57721566490153286060651209008240243104215933593992359880576723488486772677766467093694706329174674951463144724980 7\ldots^{(\clubsuit)}$

$\sqrt{2} \approx 1.41421356237309504880168872420969807856967187537694807317667973799073247846210703885038753432764157273501384623091\ldots^{(\spadesuit)}$

$\sqrt[11]{1,234,577} \approx 3.57910451145915487681532121053506309627754533481882344003994440940136200339604266694996114366514005002409 79\ldots^{(\spadesuit)}$

$\left(2^{37,156,667}-1\right) \approx 2.022544068909773355341881522631568299468466025827431829895510573605475145797581250846721390095 90\ldots \times 10^{11,185,271}{}^{(\heartsuit)}$

$\log(2) \approx 0.69314718055994530941723212145817656807550013436025525412068000949339362196969471560586332699641868754200148102 1\ldots^{(\diamondsuit)}$

$\tan\left(\frac{41}{47}\right) \approx 1.19096924080642343385499535864391655166210224629673547857304600594374722030068682468661891601859189079094775669\ldots^{(\diamondsuit)}$

$K\left(\frac{223}{227}\right) \approx 3.41631858359565670012207567527668502395404546289407743161606533879405092373789289556585886720106710302495575197 3\ldots^{(\bullet)}$

$\Gamma\left(\frac{1993}{733}\right) \approx 1.56832829296510092932380417039280885375999457346886819067779819680920995389510328229092820783838846775884559403\ldots^{(*)}$

$\psi^{(7)}\left(\frac{137}{103}\right) \approx 520.65928649004380134607822408858033860038222629342382749339468928096649221194570206982681523534169510010458092 1\ldots^{(\star)}$

$\zeta(3) \approx 1.20205690315959428539973816151144999076498629234049888179227155534183820578631309018645587360933525814619915779526\ldots^{(\ddagger)}$

$J_2\left(\frac{977}{97}\right) \approx 0.25345997611996908988882205817748010929828990811653582213814484514265655593603922525970423851794541649313648170\ldots^{(\triangle)}$

$Y_{(3,000,029/3)}(1,000,033) \approx 0.00605786233373856260719742609337254407783732009094563056212184589112167846144005691875952794032605 9\ldots^{(\triangle)}$

$P_{5/7}^{1/3}\left(\frac{467}{509}\right) \approx 1.15204352041274388176771908922289204188381679871479922114695390555455255508547453441081008957504422021540998 69\ldots^{(\circ)}$

Calculations use high-performance multiple precision floating point programs designed for numbers with about $10^2 \ldots 10^7$ decimal digits of precision. Certain integer functions and coefficients use symbolic math. **Multiplication** uses $O\left(N^2\right)$ traditional and $O\left(N^{\log_2(3)}\right)$ Karatsuba as well as $O\left(N \log_2(N)\right)$ Schönhage-Strassen FFT algorithms. **Fast Fourier Transforms** use the "Fastest Fourier Transform in the West" (FFTW) with the calculations distributed among 2^N CPU cores using parallel threads. **Integer Power** $^{(\heartsuit)}$ uses exponentiation by squaring. **Integer Root** $^{(\spadesuit)}$ uses quadratically convergent Newton iteration. **Constants** $^{(\clubsuit)}$ such as π, e and γ use Gauss arithmetic-geometric algorithms and binary splitting for computation of up to 32 million decimal digits. The calculation of one million decimal digits of π takes less than 10 seconds on a modern dual-core system. **Elementary Transcendental Functions** $^{(\diamondsuit)}$ use Taylor series, argument scaling, recursion and Newton iteration. **Orthogonal Polynomials** use generating functions and recursion. **Primes** and **Prime Factorization** use sieves and divide-and-conquer. **Elliptic Integrals** $^{(\bullet)}$ use arithmetic-geometric methods. **Gamma** $^{(*)}$ uses recursion and asymptotic series. **Polygamma** $^{(\star)}$ uses recursion, asymptotic series and Euler-Maclaurin summation. **Zeta** $^{(\ddagger)}$ uses the product over all primes, an accelerated sum of reciprocal powers, and Euler-Maclaurin summation. **Airy** uses Taylor series, asymptotic series and Bessel function representation. **Bessel** $^{(\triangle)}$ uses Taylor series, recursion, asymptotic hypergeometric series and uniform asymptotic Airy type expansions. **Hypergeometric** including **Legendre** $^{(\circ)}$ uses Taylor series, recursion and various asymptotic series. **Software design** uses Microsoft® Visual Studio® 2008, GNU Compiler Collection (GCC) 4.3.3, GNUmake 3.81, Intel® C++ 11.0.066, Mathematica® 7.0.1, the C++ programs `e_float` and `mp_cpp` (`ckormanyos@yahoo.com`), GNU Multiple Precision (GMP) 4.2.4, and FFTW 2.15. **Visualization** uses LaTeX.

[††] π Archimedes' constant; e the natural logarithm base; γ the Euler-Mascheroni constant; $\sqrt{2}$ Pythagoras' constant; $\sqrt[11]{1,234,577}$ an integer root of a random prime number; $\left(2^{37,156,667}-1\right)$ 1 subtracted from a huge integer power of 2 expressing the 46^{th} Mersenne prime number; $\log(2)$ the natural logarithm of 2; $\tan\left(\frac{41}{47}\right)$ the tangent of a rational number; $K\left(\frac{223}{227}\right)$ the complete elliptic integral of a rational number; $\Gamma\left(\frac{1993}{733}\right)$ the Gamma function of a rational number; $\psi^{(7)}\left(\frac{137}{103}\right)$ the 7^{th} order Polygamma function of a rational number; $\zeta(3)$ the Zeta function of the prime number 3, also known as Apéry's constant; $J_2\left(\frac{977}{97}\right)$ the second order cylindrical Bessel coefficient of a rational number; $Y_{(3,000,029/3)}(1,000,033)$ a cylindrical Neumann function with a very high rational order evaluated for a prime-numbered argument in the transition region; $P_{5/7}^{1/3}\left(\frac{467}{509}\right)$ a hypergeometric Legendre function with rational degree and rational order evaluated for a rational argument.