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Committee on Mathematics Education

Niveaux de référence pour l'enseignement des mathématiques en Europe

Reference levels in School Mathematics Education in Europe

National Presentation

FINLAND

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1. General description of the mathematics teaching context

1.1 Description of the country school system

The traditional long-term objectives of Finnish education policy have been to raise the general standard of education and to promote educational equality. Efforts have been made to provide all population groups and regions of the country with equal educational opportunities. These are the basic tenets of the educational reforms carried out over the last few decades. In the 1990s, however, special attention is being paid to the content of education and the methods of instruction, as well as to educational standards and equality. Increasing overall flexibility and opportunities for individual choice are also considered important; internationalization has also emerged as a key objective.

The Finnish education system consists of comprehensive school, post-comprehensive general and vocational education, higher education and adult education (Figure 1). Comprehensive school is a nine-year system providing education for all children of compulsory school age. Every Finnish citizen is required to complete this education. The school starting age is seven. Teaching groups in basic education are formed according to year classes, i.e. forms. During the first six years, instruction is usually given by the class teacher, who teaches all or most subjects. Instruction in the three highest forms is usually in the form of subject teaching, where different subjects are taught by subject teachers.

Post-comprehensive school education is given by general upper secondary schools and vocational schools. The upper secondary schools offer a three-year general education curriculum, at the end of which the pupil takes the national matriculation examination, which is the general eligibility criterion for higher education. The general upper secondary school network

covers the entire country. The schools follow a national core curriculum, but recently the range of choice has been widened. Individual schools can cultivate a more distinct image; some upper secondaries have a specialized curriculum, giving emphasis to the arts or some other field. Upper secondary school has traditionally constituted the main channel to university education.

Finnish vocational education and training is institution-based to a very large extent. Taught courses form the core of the programmes. Upper secondary vocational education covers some 75 qualifications. The study programmes are designed for comprehensive school leavers and lead to basic vocational qualifications. By the year 2001, all vocational education will be based on three-year programmes. The reform will clarify the structure of education and abolish inequalities in opportunities in further education: all three years study programmes provide eligibility for institutions of higher education. Higher vocational education leads to post-secondary and higher vocational qualifications, which take from 2 to 4.5 years to attain. Students who have passed the matriculation examination or have a basic vocational qualification are eligible for admission. The system is currently being reformed: eventually, all higher vocational education will be provided at polytechnics.

There are 20 universities in Finland, ten of which are multifaculty institutions and ten specialist institutions. Of the specialist institutions three are universities of technology, three are schools of economics and business administration, and the remaining four are art academies. In addition, university-level education is provided at one military academy under the Ministry of Defence. All universities engage in both education and research and have the right to award doctorates. The first university degree, which roughly corresponds to a Bachelor's, can generally be attained in three years of full-time study and the higher, Master's degree in five years, i.e. additional two years after the Bachelor's degree. There is also an optional pre-doctoral postgraduate degree of licentiate, which can be completed in two years of full-time study after the Master's degree. Full-time studies for a doctorate take approximately four years following the Master's degree.

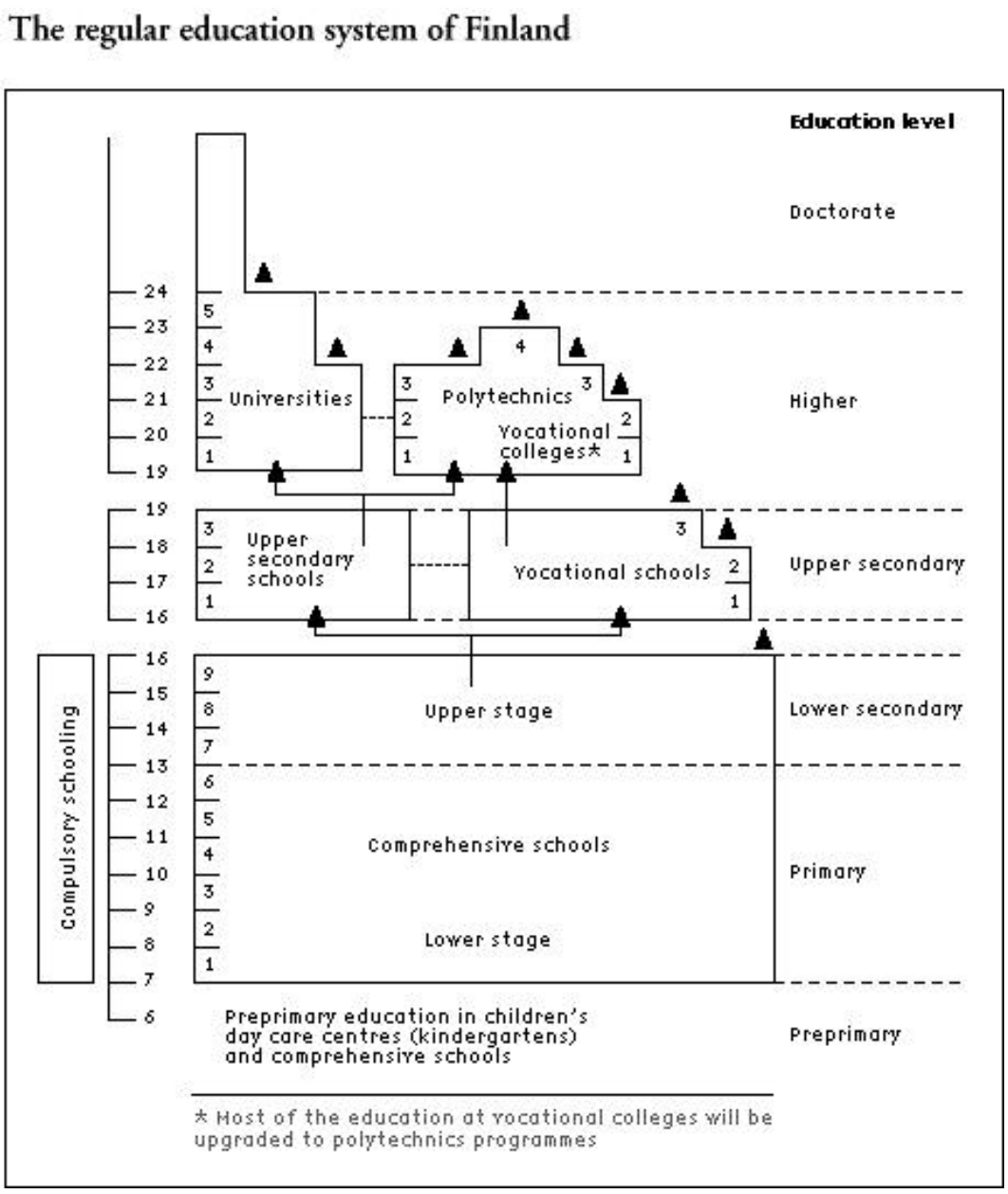

Figure 1. The regular education system of Finland.

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1.2 **Place and importance of the mathematics in the curriculum**

The basic education syllabus in comprehensive schools includes at least the following subjects: mother tongue and literature (Finnish or Swedish), the other national language (Swedish or Finnish), foreign languages, environmental studies, civics, religion or ethics, history, social studies, mathematics, physics, chemistry, biology, geography, physical education, music, visual arts, craft and home economics (Appendix 1 & 2). The broad national objectives and the allocation of teaching time to instruction in different subjects and subject groups and to pupil counselling are decided by the Government. The National Board of Education deci-

des on the objectives and core contents of instruction by confirming the core curriculum. Based on these, each provider of education prepares the local basic education curriculum.

Since 1982, instruction in upper secondary schools has been divided into courses, each consisting of about 38 lessons (lesson hour = 45 min). The school year is usually divided into five or six periods. A separate timetable is drawn up for each period, concentrating on certain subjects. Students' progress and the composition of teaching groups thus depends on the students' choice of courses. Consequently, year classes have been abolished in all upper secondary schools, which now function without fixed forms.

Upper secondary school studies consist of compulsory, specialisation and applied courses (Appendix 3). All students must complete the compulsory courses. Schools must provide specialisation courses for students to choose from. Each student is responsible for completing a sufficient number of courses. Applied courses may be either further studies in subjects already studied or other subjects. The provision of these courses can be decided independently by each school. They can also be offered in co-operation with other educational institutions, such as vocational or music institutions.

In upper comprehensive schools (ages 13-16), the level of studies in mathematics is the same for all pupils. Instead, in upper secondary schools (ages 16-19), every student chooses either short or advanced level courses in mathematics including compulsory and specialisation courses. The weekly number of the lessons in mathematics varies a lot (from 0 to 6 lesson hours per week), depending on the level of studies and the period of the school year. However, the main difficulty in determining a relevant average time for studies in mathematics per week is due to local basic education curriculums prepared by the each provider of education. The rough estimates of the studies in mathematics per week is about 3 hours in upper comprehensive schools and 2.7 hours (short level) or 4.3 hours (advanced level) in upper secondary schools. The total number of lessons per week is about 30 hours.

The matriculation examination consists of four compulsory subjects and one or more optional ones. Candidates may take it in three consecutive examinations, that is, over a period of 18 months. The compulsory tests are in the mother tongue (either Finnish or Swedish, depending on the language of instruction at the school), the second official language, a foreign language, and either mathematics or general studies. In the last-mentioned examination, the student answers questions in one or several subject groups, which are: religion and ethics; psychology and philosophy; history and civics; physics, chemistry, biology and geography. There are two levels of examinations in mathematics, in the second official language and in foreign languages; in at least one of the compulsory examinations the more demanding level must be chosen.

2. Main mathematics objectives

Framework curriculum for the comprehensive school 1994 (National Board of Education 1994a) identifies the main mathematics objectives to be achieved during the compulsory schooling (ages 7-16).

During the comprehensive school all students are given the opportunity to get such basic mathematical knowledge and skills that create a foundation for further studies and prepare them to get along in everyday life and in working life. The aim for the comprehensive school mathematics is most of all to develop the student's ability to classify, organize, and model situations that come up in the surrounding world, with terms he has learned.

Along with this, the aim is to train students to engage in logical and exact thinking and to present things verbally as well as in writing. Students should also have an understanding of the importance of mathematics in the past and present and its part in the development of our culture.

As regards contents, comprehensive school mathematics must have an open mind to new information, inventions, matters that have gained in importance, and current applications. Even though in choosing and organizing the syllabi the origins of mathematics should be taken into consideration, traditional contents must be reviewed critically. Such information that is not necessary in understanding mathematical structures and applications should be left out. On the other hand, broader syllabi than before must be formed so that the students have a better chance of understanding mathematical terms and information structures. In this way the role of the mechanical counting can also be reduced at all levels.

At the end of the lower stage, things learned earlier are expanded by organizing information, and this is when most students have the chance to form systematic wholes from the information they have acquired. At the same time, the knowledge of basic information and skills is reinforced.

In mathematics, the student is seen as one who actively gathers, handles, and saves information, and as one to whom learning is the same as combining what he is taking in with what he learned earlier and reorganizing and complementing earlier thought and activity models. The purpose here is to have the student – as a result of long-term work – gradually organize knowledge and skills in his mind into a useful structure.

The learning situations should be in the form of discussions, experiments, and problem-solving, and the students should as often as possible be given concrete everyday problems to solve. Right from the outset in mathematics, the aim is to understand concepts. It happens through concrete activities and through and emphasis on crafts and games. The numerical expressions of numbers and the basic calculations are slowly introduced – and then only through the study of practical problems, students' verbal interpretations, and measurements.

Solving problems is a central principle along with mathematical-logical requirements. The meaning of problem-solving processes is emphasized, above all, in gathering information, and also in the application thereof. Asking pointed questions, getting an idea of the problem, limiting the problem, finding and implementing proper solution methods and evaluating and formulating results are important in learning.

Students of all ages and at all levels should be allowed to build and make models with their hands in order to form correct mental pictures and concepts. This facilitates the development of logical deduction skills. Mathematical thinking is best supported by not moving too quickly to abstract symbolism. Calculators and computers should be used as natural aids from the

lower level onwards. This gives students a chance to understand the reason for mathematical knowledge and gives them confidence in their own abilities to build, learn, and use Mathematics in meaningful and reasonable situations.

In order to have the students learn concepts and structures of knowledge, it is important to plan more extensive wholes out of single contents or important concepts. When things are learned as extensive wholes, the haste in teaching is reduced and we avoid teaching things over and over again. For example, in learning fractions, it makes sense to learn the concept of percentage. Decimal numbers in connection with measures, transformations of units of measure, and the deepening of the percentage concept form one important whole. At the upper level, similar wholes are negative numbers, the systematic treatment of numbers being raised to a power, direct proportionality, matters that have to do with dependency, congruence, and trigonometry, and some environment where the students prove theorems.

In order for the students to get a realistic picture of the usefulness of mathematics, the teaching must be integrated with other work in school as well as with the outside world in different ways. In this manner, the students get experience in blocks of knowledge offered by Mathematics, in the use of models, an in their limitations. For example, interdisclipinary projects are a good way to cross subject boundary lines at all class levels.

Mathematics arouses strong feelings in people, both in adults and children. Learning experiences and expectations as well as the setting and its implementation often determine the direction these feelings take. Motivation for study makes it necessary to offer good opportunities to study and arouse a positive, inner desire to learn. This is enhanced by choosing the examples from the student's life and through practical work. This adds to the attractiveness, excitement, and surprisingness of mathematics. On the other hand, the students must show initiative, cooperation, and originality, and it could also be toilsome and require perseverance.

Framework curriculum for the senior secondary school 1994 (National Board of Education 1994b) gives the corresponding framework for the upper secondary school (ages 16-19). This framework sets up the general goals. However, it somewhat lacks the detailed specification. The same applies in the case of the comprehensive school curriculum.

The purpose of teaching of the advanced course in mathematics is to offer students an opportunity to acquire the knowledge and skills that are needed especially in the study of mathematics, the natural sciences, and technology both in vocational and university studies. The aim is to give students a sense of mathematics as a developing discipline, some insight into the structure and theory formation of mathematics, and some idea of its significance and application possibilities in everyday life, science, and technology.

The purpose of teaching of the short course in mathematics is to develop students' general civic abilities in the acquisition, treatment, and understanding of mathematical knowledge, as well as in the use of mathematics in various life situations. The subject aims at providing abilitiesa for continued studies primarily in the fields of the humanities, the social sciences, and economics.

The purpose of the study of advanced mathematics is that the students

- learn to appreciate and understand the role of mathematics in social development and decision-making, as well as in people's everyday lives;
- learn to trust their own mathematical abilities, skills, and thinking, and learn perseverance; for this purpose, they are encouraged to engage in experimental and analytical activities, in finding solutions as well as in their critical assessment;
- learn to understand an use the language of mathematics, to follow the presentation of mathematical information, to discuss mathematics and read mathematical texts, as well as to appreciate exactness and clarity of presentation;
- develop, through many-sided practice, the ability to use calculation and reasoning skills, various aids and reference materials;
- know how to use and apply mathematics to solve problems, learn to construct models in vrious practical problem situations, and to utilize different solution strategies; and
- be able to handle information in the way that is typical of mathematics, learn to make assumptions, study their correctness and give exact reasons, as well as learn to assess the validity of reasons that have been presented and the generalizability of the results.

The purpose of the study of the short course in mathematics is that the students

- know how to use mathematics as a necessary aid in everyday life situations and in solving problems in society;
- get positive learning experiences and learn to trust their own abilities, skills, and thinkding; for this purpose, they are encouraged to engage in experimenting, investigating, and innovative learning;
- acquire useful mathematical knowledge, skills, and abilities that form a sufficiently good basis for continued studies;
- realize the significance of mathematics as a means in describing, explaining, and making models of reality, and its use in drawing conclusions;
- develop their ability to analyze mathematical knowledge and to understand its logical structure; and
- learn, as part of their general education, to receive, analyze, and critically assess information offered in mathematical form by the mass media.

3. Basic contents

The first part of this section concerns the basic contents of mathematics studies in the comprehensive school (ages 7-16) (National Board of Education 1994a).

At the lower stage of comprehensive school it is important that the student

- learns to perceive the world around him and to interpret it in mathematical terms and to recognize problem situations and how to act in them;
- understand the concepts of a natural number, fraction and decimal and can do basic calculations in his head, on paper, and on a calculator, and uses them in everyday problem-solving and gets used to evaluating the scale and correctness of results;
- learns to estimate and measure lenghts, mass, area, volume, angles, and time, and learns the most common units of measurement for them and how to transform them;
- understands the term scale and learns to use it in interpreting drawings and maps;
- learns to recognize and draw the most common geometric shapes and patterns, to describe their basic characteristics, and to calculate their area, and volume and to familiarize himself with symmetry;
- familiarizes himself with sorting and classifying things and objects, with detecting regularities in the surrounding world and with describing them as well as with produ ing, reading, and interpreting simple charts and diagrams.

At the upper stage of comprehensive school it is important that the student

- gets a picture of real numbers, learns to calculate using real numbers in everyday situations and is able to estimate the scale of results and whether the results are sensible;
- know how to collect, chart, interpret, and use statistics, and understands and knows how to use the concept of probability;
- is able fo find regularities and dependencies in the surrounding world and learns the use of the variable and the concept of function in expressing these; furthermore, the recuacion of expressions and the use of coordinates in charting illustrations;
- is able to make mathematical models out of everyday problems, can use equations and pairs of equations and handle letter-form expressions in connection with solving problems and modeling, and learns to interpret the results of the solutions;
- understands the basic concepts in geometry, knows how to draw the most common patterns and solids and is used to using proportionality, trigonometry, and the Pythagorean theorem in calculating areas and volumes of patterns and solids;
- understands the concepts of congruence, similarity, and symmetry and sees their usefulness, familiarizes himself with the importance of deduction in mathematics and with how

structures are formed in mathematics, and gets a picture of how knowledge gleaned in the comprehensive school can be formed into a systematic whole.

The next part of this section concerns the basic contents of the compulsory (advanced and short) courses in mathematics in the upper secondary school (ages 16-19) (National Board of Education 1994b).

Advanced course in mathematics

- **Functions and equations I.** Students are taught to study mathematical models that describe the characteristics of real phenomena so that they recognize dependencies as well as assumptions and simplifications that are presupposed by the model. They learn to use mathematical methods in problem-solving and to interpret the results critically. Functions are studied on the basis of graphic representation, and mainly polynomial and exponential functions are examined by means of graphs, equations and inequations, and modern tools for calculation and graphics. The concept of the function is clarified through many-sided exposure thus building the foundation for the theory of functions that is central in mathematics.

- **Functions and equations II.** Students continue to deal with polynomial functions and equations. They study the divisibility of polynomials and the characteristics of the roots of polynomial equations. They practise using, interpreting and solving inequations. Students develop their skill in using mathematical models in problem-solving, in drawing conclusions and giving reasons, and in assessing the validity of results. The study of functions is expanded to cover other algebraic functions and logarithmic functions. Students use graphic and calculation aids that enhance understanding so that learning is closely linked to many-sided applications.

- **Geometry.** Students are taught to make exact observations and conclusions about the surrounding world, and to classify and give reasons for the features that are linked with figures. Spatial perception is developed through the study of three-dimensional objects. Different projections and intersections are made on the basis of these objects, and calculation and graphic problems are solved by them. Appropriate computer programmes and construction models are used, if possible. The course clarifies some characteristics of geometric descriptions, and the students practise giving proof, for example, with regard to congruence and similarity.

- **Analytical geometry.** Students learn about analytical geometry. Its importance is emphasized in the creation of connections between geometric and algebraic concepts. Students

study points, lines, circles, and parabolas in the coordinate, they learn about the use of parameters, and expand the study to cover three-dimensional cases, too. Students solve equations and inequations with two variables, and learn about linear optimization.

- **Trigonometry and vectors.** The course presents trigonometric functions and their basic properties as well as their use in the most common applications. It also introduces the concept of vector, its basic properties and computations, as well as giving practice in using two- or many-dimensional vectors in various contexts.

- **Differential calculus I.** The course deals with concepts of limit, continuity, and derivative, as well as their basic properties and various application possibilities. Students are taught to internalize the concept of derivative and they become accustomed to using the derivative in many ways in the study of polynomial an algebraic functions and in problem-solving.

- **Differential calculus II.** The functions studied are expanded to cover transcendental functions whose basic properties, derivatives, and usage are examined. Mathematical analysis is applied in the study of functions as well as in the solving of equations and inequations. Students examine the application of the theory in practical situations.

- **Integral calculus.** Students are introduced to the concepts of integral function and definite integral as well as their definition. They study how to determine areas and spaces by using integration. They also become familiar with other applications of integral calculus and with methods to calculate integrals numerically with the help of calculators and computers.

- **Statistics and probability theory.** Students are introduced to the different stages of statistical analysis. They are taught to construct and interpret statistics, to assess conclusions drawn from them critically, and to understand the uncertainty that chance renders to statistical data. They are introduced to the concepts of classical and statistical probability, computation rules pertaining to them, and to applications of probability theory. Students learn, on the basis of empirical data, about various distributions and ways to describe their properties. They use calculators and computers in many ways. They also study topical statistical in formation offered by the mass media, and work in projects.

- **Sequences and series.** Students are taught to study and use the basic properties of natural numbers, including their property of induction. They study sequences and series, and apply them to the solution of practical problems. According to resources and interests, the

course can be expanded to cover, for example, the use of differential equation, graph theory, algorithms, or coding.

Short course in mathematics

- **Statistics and probability theory.** Students learn how to interpret and critically analyze statistics, and also how to assess the conclusions and their correctness. The course increases the skills of mastering knowledge. The students learn about different stages of statistical analysis from collecting the data themselves to its graphic representation and interpretation. Dependencies between statistical variables are examined through calculations and graphics. The other main theme of the course is probability theory including the basics of probability calculations, rules governing the calculations, and the use of binomial and normal distribution in applications. In the course, calculators and computers are used frequently; it is also typical of the course to analyze topical statistical information that is presented by the mass media.

- **Mathematical problem-solving.** Students learn about the significance of mathematics in the development of culture, and about numerical relationships and principles. They learn to use mathematics in solving everyday problems and problems of their own so that they learn to rely on their own mathematical abilities. The students become accustomed to expressing dependencies between quantities through expressions, to formulate problems in the form of equations, to solve them graphically and algebraically, and to interpret and assess the solutions.

- **Geometry.** Students learn to perceive two- and three-dimensional space and to draw plain figures and pictures of three-dimensional objects. They learn about the properties of these pictures through calculations an through computers. They learn to integrate visual perception as part of the study of mathematics. Geometric graphs are dealt with. Students practise solving practical problems with the help of the similarity principle, trigonometry, and the Pythagorean theorem. Students become familiar with the use of coordinates in geometrical contexts.

- **Mathematical models.** Students learn to realize the significance of and need for mathematical models in real situations and they get familiar with various possibilities to make models of real world phenomena. The course deals with dependencies between variables and students learn about the most common functions. They learn to put into practice linear and exponential models, and also to use polynomial and exponential equations and logarithms. They also get accustomed to assessing the conditions, limitations, and use of the models.

- **Mathematical analysis.** Students are introduced to the concept of a derivative as a measure for the rate of change using graphic and numerical approaches. Students learn about the use of f derivative in the examination of changes, and they learn how to determine the biggest and the smallest value using both the derivative and other methods according to practical situations. Students become familiar with graphic and numerical approximation methods.

- **Mathematical research methods.** In this course, various mathematical research methods are compiled and complemented, and they are applied to various types of tasks. Students learn to examine number sequences and sums in application situations. They learn to use various functions as mathematical models and study them by means of equations, graphic and numerical methods, and a derivative. The course strives at forming extensive application entities, if possible. The course emphasizes the position and significance of mathematics as a tool for other subjects, disciplines, and society. The applications are selected from various everyday situations in the fields of trade and economics, the environment and nature, and the individual and society.

The study of mathematics in the upper secondary school can be deepened either by studying a familiar subject area in more detail or by learning about a totally new field of mathematics. In the advanced course in mathematics, specialization courses can be planned to cover some of the following:

- **analysis**, where the number of functions is expanded, methods of integration supplemented, functions with two variables, partial derivatives and differential equations introduced, and clusters of curves studied;
- **numerical methods**, which are realized with the help of calculators and computers. This course introduces solution algorithms and approximation methods in the solving of equations and equation groups as well as in differential and integral calculations;
- **number theory and logic** where the students learn about Boolean algebra, the basics of logic, and the algebraic structures of number areas.

Other possible topics include probability calculations, statistical reasoning, and economics.

In the short course in mathematics, specialization courses can be planned to cover some of the following:

- in **econometrics** students learn about calculations related to indexes, costs, money transactions, loans, taxation, etc. Both from the point of view of the individual and of society. Mathematical models that are suitable in economic situations are developed on the basis of number sequences and series. Statistical methods are applied especially to the analysis of self-collected data or to project tasks, and a picture is formed of the mathematical ma-

nagement of risks and profitability. Students learn about the principles of linear optimization. The course offers possibilities for the study of entrepreneurship and economics;

- **probability and statistics** may be dealt with in a course especially designed for the students of the short course in mathematics or they may participate in a corresponding course for the students of the advanced course in mathematics.

Other possible topics include vectors and analytic geometry as well as an advanced course of analysis.

4. Exemplary topics

The national curricula for the comprehensive school and the upper secondary school (National Board of Education 1994a & 1994b) do not specify the exact contents or the year classes when various subjects are to be introduced. It is therefore impossible to present exactly when and how the following examplary topics are taught. The estimated times and contents mentioned here are based on the presentation of these topics in a book serie by a Finnish publishing company (Metiäinen et al. 1996 & 1998 & 1999). The examples of the theoretical approaches and exercises below are also taken from this book serie. The textbooks are written in Finnish, and the examples have been translated into English. More examples can be found in the Website http://www.mansoft.fi/manmath/Kokonaisuudet/Peruskoulu/Peruskoulu.html, which contains a repetition of the syllabus in mathematics in the comprehensive school (Mansoft tietotekniikka Oy 1999).

4.1 Quadratic equations

The quadratic equation is a topic in the upper secondary school (ages 16-19). The general quadratic equation is given in the form $ax^2 + bx + c = 0$, and the students learn the formula for solutions and the role of the signs of the discriminant. However, special cases of the quadratic equation are solved graphically during the last year class of the upper comprehensive school (ages 15-16).

Example 1. **(**Metiäinen et al. 1999, Koontaa ja syventämistä, p. 58)

Example 1 Draw the graph of the function $f(x) = x^2 - 4$.

Example 2 What are the zeros of the function of the example 1?

4.2 Pythagorean Theorem

Pythagorean theorem is a topic in the upper comprehensive school (ages 13-16). The main contents of the course of geometry in the last year class are the Pythagorean theorem. The student is supposed to be used to using it in calculating areas and volumes of patterns and solids. The theorem is presented in the form $a^2 + b^2 = c^2$.

Example 2. **(**Metiäinen et al. 1999, Geometriaa 3, pp. 14-15)

Pythagorean theorem

Every right-angled triangle satisfies the following theorem:

> If the lengths of the two shorter sides are squared and the results are added, the length of the hypotenuse squared is obtained.
>
> On the other hand, if the lengths a, b and c of the sides of a triangle satisfy the condition $a^2 + b^2 = c^2$, then the triangle is right-angled.
>
> **Exercise 9.** Is the triangle right-angled if the lengths of its sides are
>
> a) 4, 5 and 6 c) 25, 7 and 24
>
> b) 80, 18 and 82 d) 4, 12 and 8.

4.3 Similarity

One of the objectives in the lower comprehensive school is that the pupil understands the term scale and learns to use it in interpreting drawings and maps. Another objective is that the pupil learns to recognize and draw the most common geometric shapes and patterns, and to familiarize himself with symmetry. In the proper sense of the similarity, the concept is supposed to be understood during the upper comprehensive school (ages 13-16). Moreover, similarity is a content of both short and advanced level courses in geometry in the upper secondary school. The advanced course clarifies some characteristics of geometric descriptions, and the students practise giving proof, for example, with regard to congruence and similarity.

Example 3. **(**Metiäinen et al. 1999, Geometriaa 3, pp. 30-31)

Similar figures

Two figures are similar if

- the corresponding angles are equal
- the ratios of the lengths of the corresponding line segments are equal.

The ratio of the lengths of the corresponding line segments is called the scale.

If one wants to prove that two figures are similar, one must show that the both conditios above are valid.

4.4 Word problems

Word problems, generally speaking, have a central role when the pupil learns to perceive the world around him and to interpret it in mathematical terms and to recognize problem situations and how to act in them. This is one of the main contents of school mathematics in the comprehensive school, starting from the first year class (age 7). In the upper comprehensive school, the student is supposed to be able to make mathematical models out of everyday problems and to handle letter-form expressions in connection with solving problems and modeling. Of course, solving of word problems continues in the upper secondary school.

Example 4. **(**Metiäinen et al. 1996, Laskuja ja laskutoimituksia, p. 9)

Exercise 2. Five floorball teams participate in the tournament. Every team plays once against every other team. How many matches are played?

4.5 Percentages

Some easy cases of percentages appear already in the lower comprehensive school. Principally, percentages are included in the contents of mathematics in the upper secondary school (ages 13-16). Associated with percentages, the students in the upper stage learn how to interpret and use statistics, and understand and know how to use the concept of probability.

Example 5. **(**Metiäinen et al. 1998, Prosenttilaskentaa ja potenssioppia, pp. 30-31)

How many percent is something changed (increased or decreased)?

- ***The change is calculated first.***
- ***Then it is calculated how many percent the change is from an original value:***

A quotient is converted into percent

Change/ original value

Exercise A2. There were 964 inhabitants in Hailuoto at the end of the year 1992. At the end of the next year, there were 983 inhabitants. How many percent had the number of inhabitants increased?

4.6 Functions (an additional topic)

In the upper comprehensive school (ages 13-16), the student learns the use of the variable and the concept of function. It is important to understand the use of coordinates in charting illustrations. The study of functions is continued in the upper secondary school, and the concept of the function is clarified through many-sided exposure thus building the foundation for the theory of functions that is central in mathematics.

Example 6. **(**Metiäinen et al. 1996, Tilastoja ja funktioita, p. 55)

example 1 *Put spots to the* xy*-coordinate system such that the coordinates of the spots satisfy the following condition: the sum of the* x*-coordinate and the* y*-coordinate is equal to 2.*

Other subjects interest for mathematics education around 16 year old

5.1 Regional characteristics

As mentioned before, each provider of education in Finland prepares its own basic curriculum based on the broad national objectives. The national distribution of lesson hours does not specify the year classes when various subjects are to be introduced. Neither does it set upper limits to the number of hours the subjects have. The distribution of lesson hours for various students can therefore differ between different schools and even within the same school. Moreover, comprehensive and upper secondary schools can develop individual profiles by focusing on some area, such as languages, mathematics and sciences, sports, music or arts.

Swedish-speakers (6 % of the population) mainly live in coastal areas in the south and west, as well as in the self-governing Province of Åland. There is a separate school system for the Swedish-speakers comprising some 330 comprehensive schools and over 30 upper secondary schools.

In Finland, there are still regional differences in education due to some non-administrative reasons: the proportion of people aged 15 or more who have at least a degree at the upper secondary level varies locally from 50 percent to 60 percent, while the rate for the whole country is 55 percent. Social background still affects educational choices: children of blue-collar workers and farmers tend to opt for vocational education, whereas the children of white-collar workers usually go to university. However, the rising education level of parents should gradually affect their children's choices: the more highly educated the parents are, the more willing their children usually are to obtain a higher qualification.

5.2 Implementation strategies

Education forms a vital part of the Finnish strategy for promoting citizens' well-being, cultural wealth, sustainable development and economic success. The foremost overall principles in educational development are high quality, equal opportunities and lifelong learning. An effective and efficient innovation system, with balanced funding, is indispensable for the development of the economy and employment based on innovations.

Under the Decree on Education and University Research of 25 January 1991, the Government adopts a plan for the development of education and university research every fourth year for the ongoing and five calendar years. On 21 December 1995, the Government adopted the development plan for the period 1995-2000 (Ministry of Education 1999a). It also contains an overall plan for language instruction in comprehensive schools.

The measures approved for 1994-1996 in the revised (Government resolution, 18 June 1993) plan for 1991-1996 have been initiated and for the most part implemented. The long-term objectives for education and research mentioned in the revised plan are still valid.

Comprehensive schools

- The municipal school system will continue to form the backbone of compulsory education. The resources for the comprehensive school will continue to be secured through the public financing system.
- The distribution of classroom hours and the curricular guidelines currently in force will be applied during the whole planning period.
- The administrative demarcation between the lower and the upper stages of the comprehensive school will be eliminated and conditions will be secured for teaching independent of year-classes.
- Pupil assessment in the comprehensive school will be diversified. An inquiry will be made into the problems in end-of-school assessment and joint selection to further education and training, and the necessary reforms will be introduced in order to secure pupils' legal rights.
- Care will be taken to ensure cost-effectiveness in the school network and access to education.

Upper secondary schools

The distribution of classroom hours and curricular guidelines currently in force in the upper secondary school will be applied during the whole planning period. The matriculation examination will be further developed, for instance by means of more diversified test assignments.

Cooperation between upper secondary schools and vocational institutions will be intensified with a view to enabling young people to make optimal use of the educational provision available. Individual study programmes will be encouraged. Students are offered opportunities to combine studies in different institutions in their programmes.

Improvements in mathematical and scientific know-how will be launched on the basis of the development programme of the National Board of Education to raise the educational attainment of young Finns to the top quarter of the OECD countries. The proportion of mathematics and sciences will be increased in the training of class teachers.

The education system will be developed toward openness, so that a student always has access to flexible pathways to further studies. Measures will be taken to secure opportunities for non-matriculated students to progress to the non-university and university sectors.

5.3 Teacher training

All education and training of comprehensive and upper secondary school teachers was transferred to the universities between 1973 and 1975. Eight universities have a Faculty of Education for pedagogical research and education. Teaching practice takes place partly at university-run training schools (13) and partly at municipal comprehensive and upper secondary schools. The aim is to raise attainment standards and enable teachers to make use of research findings to serve the school and to improve their own teaching performance.

Class teachers teach the lower level of comprehensive school (school years 1-6). The class teacher's degree is a higher university degree, i.e. a Master's degree, based on a programme comprising 160 credits. Students major in education, and the programme also includes basic studies in several subjects, specialist studies in one or two subjects, and teaching practice. The class teachers' degree programme is one of the most popular fields of study offered by the universities. The standard of applicants is high, and only 10-15 percent gain admission.

One important question has turned out to be the share of mathematical and science subjects in the degree requirements of class teachers. Since class teacher trainees do not often take minor subjects relating to mathematics and sciences, more attention should be paid to the quantity and quality of these subjects in the multidisciplinary studies in the subjects and integrated themes taught in the comprehensive school.

Subject teachers teach one or more subjects at the upper level of comprehensive school (school years 7-9, i.e. lower secondary level) or at upper secondary school (school years 10-12). They take the higher university degree in the faculty responsible for research and instruction in their major subject. The degree programme for prospective teachers differs from others mainly in that 35 of the 160 credits required for the degree consist of teacher's pedagogical studies offered by the Faculty of Education. This block of 35 credits includes teaching practice, which can be completed after graduation. New school legislation coming into force at the beginning of 1999 will remove the administrative division between the lower and upper levels of the comprehensive school.

The training of remedial teachers includes a major subject, remedial pedagogy, optional minor subjects, pedagogical studies for teachers and remedial teacher studies. The general aim in teacher education and training is to see that the quality and quantity of education match the needs of the school system. A topical issue is the harmonization of forms of education and qualification requirements to enable teachers to move easily from one speciality to another, and to make it possible for different types of schools to share teachers. The Higher Education Evaluation Council is carrying out an assessment of teacher training in 1998-1999.

5.4 Resources available to teachers

MAOL (The Mathematics and Science Teachers Association) assists its members at their pedagogical work and also backs up the work done by its local associations

- by giving out a magazine *Dimensio* six times a year; the magazine contains current information about science at school and what is new in the field of teaching it,
- by mailing a member bulletin four times a year telling about happenings, courses and further education,
- by arranging further education twice a year and participating in the arrangement of the ITK Congress,
- by arranging courses on current topics,
- by arranging national competitions in science for the pupils of the comprehensive schools and the upper secondary schools,
- by maintaining international connections with pedagogical organisations and arranging study and congress trips,
- by actively participating in educational politics both locally and nationally.

MAOL owns together with the local clubs a publishing company *MFKA Publishing inc.* to produce and supply

- computer programs and equipment,
- material for teaching and testing pupils at various levels of school.

National Board of Education provides a lot of resources for education and training in Finland. Most of the Internet services and resources are in the two official languages:

- http://www.edu.fi/oppimateriaalit/ (in Finnish) or
- http://www.edu.fi/svenska/oppimateriaalit/ (in Swedish).

Some of the services are provided also in English or include a description in English

- http://www.edu.fi/english/resource.html .

5.5 Problems already detected

One of the most serious shortcomings in mathematical and scientific knowledge is that the mathematical studies of Finnish pupils progress more slowly than the international standard. Another serious problem is that the number of subject teachers who have majored in mathematics or science is too small compared to the estimated need. Here is a short list of some specific problems:

- too large teaching groups,

- the teacher of a teaching group changes too often,
- problems associated with the course-oriented curriculum; the learning process is not continuous,
- the salaries of teachers are not competitive with the salaries of private sector companies.

The Ministry of Education's LUMA[1] programme of Prime Minister Paavo Lipponen's First Government (1995-99) included measures for raising mathematical and scientific knowledge to international standard. In 1995 the National Board of Education launched a development programme for mathematics and sciences for 1996-2000. The Ministry of Education expanded the programme to partners outside the school system and to this end devised an extensive programme (LUMA) for 1996-2002.

An intermediate evaluation of the LUMA programme was carried out in 1998. As yet, there is little internationally comparative data on the level of Finnish schoolchildren's knowledge, but the situation does not look especially good in the light of the existing data. One of the objectives was that in the matriculation examination, over 16,000 canditates will take the advanced test in mathematics; there is still long way to go.

5.6 Data on national and regional results from different tracks

In the autumn 1991, the National Board of Education began a wide and long-term development program in the basics of the curriculum which development was decided to follow regularly through connecting research to the unofficial examination realized by the teacher organization (Pehkonen 1997). The nation-wide test for the 9th grade was arranged on April 25, 1995. Its point of emphasis was: mathematics in everyday life. The test contained a mental part which consisted of mental calculations without and with a pocket calculator (in both ten tasks), and a part of word problems. There was time for the test altogether 90 min of which 30 min were reserved for mental calculations.

In the research connected to the nation-wide test, several types of data gathering were applied. Besides the test results, the pupils were given a questionnaire which concentrated on clarifying the affective factors which are combined in mathematics learning. In teachers' questionnaire, practical organization of teaching as well as teachers' conceptions on mathematics and pupils' assessment were investigated. The reliability of these indicators was checked by interviewing from five classes the teacher and a group of pupils. In the sample, there were 50 at random selected schools to cover the whole of Finland. Complete results were received from 44 schools (response percentage 88%). Thus, the group of test subjects was compound of 739 pupils, of whom 363 were girls and 376 boys, and their 44 teachers.

[1] "LUMA" is an acronym of the Finnish words meaning natural sciences and mathematics.

When investigating the influence of some backgroung factors, significant differences were observed between the pupils from the capital area and from the other cities. The amount of the teachers' career experience gave a statistically very significant difference in the case of the word problems; pupils in the classes of younger teachers reached significantly better results.

Boys gained in the nation-wide test better results than girls, especially in mental calculation without a calculator and in word problems, although girls had better school grades in mathematics than boys. Girls experienced mathematics more difficult than boys. Boys showed stronger self-confidence in mathematics than girls.

When comparing the results with the corresponding research done in 1993, the following observations were done: In the use of different supportive actions, a statistically very significant decrease was observed. In the case of pupils' explanation and the use of larger tasks, there was seen statistically significant positive development. Pupil assessment was changed in quality.

Mathematics school achievement of Finnish secondary school pupils within an international frame was clarified through comparing the level of eight-graders' (14-year-olds) skills in mathematics with their mates in some other countries in the tests of the Kassel project (Soro & Pehkonen 1998). In Finland, the tests were realized in fall 1995 in 25 at random selected classes. From the 16 Kassel project countries, the following five countries were, in addition to Finland, in this comparison: Germany, Greece, Hungary, Norway, and England.

When considering the different subdomains, Finns were still near the international average only in arithmetic. In algebra, the rank of Finland was changed from the international average to the last place of the comparison countries. In the subdomain of geometry and functions, Finns were earlier in good international average, but based on results of this study they are clearly left behind from the international level. In algebra, geometry and functions, the skills of Finnish eight-graders corresponded the skills of one year younger pupils in other countries.

The results of the Kassel project tests showed that Finnish pupils had clear deficiencies, especially in knowledge of concepts, rules and solution procedures, compared with the results of the comparison countries. But in reasoning and in problem solving, Finns were as good as pupils in the other countries. The tasks of this study measured, perhaps, more quantity than quality of mathematical skills, and thus the results reflect more the bredth of the implemented curriculum in different countries than the depth of pupils' mathematical thinking and skills.

The doctoral thesis by P. Kupari (Kupari 1999) examines the beliefs that Finnish comprehensive school mathematics teachers hold about mathematics and about learning and teaching it, and the important role that these beliefs play in the teaching of mathematics. The study is linked with a subject field that has been discussed animatedly on an international level for more that ten years. However, the present study looks for answers to the questions it addresses specifically from the perspective of Finnish teaching culture. In this way the study attemps to describe and understand the instructional practices of mathematics teachers and the preconditions under which these practices develop and chance.

The findings of the study showed that mathematics teachers hold relatively strong beliefs about mathematics. These beliefs emphasise particularly the importance of thinking and of setting pupils tasks involving problems that give them scope for independent reflection and encourage them to look for a number of potential solutions. Both the class teachers and the subject teachers had multidimensional belief structures: their beliefs were based on several different factors rather than on one single type of belief. All belief structures featured an emphasis on practical axercises, something that may be considered an aspect of the traditional idea of mathematics and of traditional teaching methods. On the other hand, another prominent ingredient of the teachers' beliefs was learner orientation, a special focus of the constructivist conception of learning. The simultaneous prevalence and vigour of such different or even conflicting beliefs is made possible by the fact that belief systems allow for the existence of competing beliefs.

Despite being multidimensional and in part contradictory, the teachers' mathematical beliefs were quite stable and changed little over the course of five years. A very obvious factor underpinning this constancy are such core beliefs as faith in a mastery of basic computational skills, in a great deal of practical exercises and in an exact use of mathematical language, which are deeply rooted and seem tenacious. No very strong links were found between beliefs and teaching practices. The teachers' beliefs were transmitted to their pupils' learning outcomes in the sense that strong beliefs concerning the centrality of pactical exercises led to a deterioration in the pupils' learning outcomes.

The National Board of Education surveyed the teaching premises and equipment of the pilot schools and the teachers' qualifications in autumn 1996. In spring 1996 the Mathematics and Science Teacher Association (MAOL) together with the Confederation of Finnish Industry and Employers (TT) studied the present situation in mathematics teaching at the primary and secondary levels to find out the development needs (TT & MAOL 1996). Both studies revealed considerable shortcomings in facilities.

5.7 Examples of inspiring activities

One topic of great interest has recently been schoolchildren's afternoon activities, including school clubs. The Centre for School Clubs assists schools in arranging clubs. The Science Education Association has arranged science clubs, projects and educational material for comprehensive school pupils. Clubs are also run by village committees, societies, business enterprises, parishes and associations.

Universities, the Economic Information Office, the Confederation of Finnish Industry and Employers and other organisations arrange various pilot activities and projects, science clubs and science teaching projects.

The education of mathematically talented students has been a focus of interest in recent discussions. It is important to identify and support talent from the outset. Päivölä Folk High School in Valkeakoski has started a pilot project in teaching mathematically talented students, as well as other projects targeted at talented students in 1994-98.

An Internet magazine in school mathematics called *Solmu* (http://www.math.helsinki.fi/Solmu/) was founded in 1996 by mathematicians at the University of Helsinki. It is given out three times a school year. The main purpose of the magazine is to offer mathematical problems and articles for the students and school teachers interested in mathematics. Moreover, it contains a feedback Website on Internet. Collaboration with Hungarians has been activated, for example, translated mathematical problems are published via Internet in *Solmu*.

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Appendix 1

TIME ALLOCATION IN THE LOWER COMPREHENSIVE SCHOOL (FORMS 1-6)	
Subject	**Compulsory courses (minimum)**
Mother tongue (Finnish or Swedish)	32
Foreign language · Language A (begun at the lower stage) · Optional language	 8 4
Mathematics	22
Biology, Geography, Environmental studies, and Civics	15
1. Religion / Ethics	8
History	3
Arts and practical subjects · Music · Art · Craft · Physical education	44 6 6 8 12

The Council of State Decision on the timetable in the comprehensive school (Helsinki on September 23, 1993).

Appendix 2

TIME ALLOCATION IN THE UPPER COMPREHENSIVE SCHOOL (FORMS 7-9)	
Subject	**Compulsory courses (minimum)**
Mother tongue (Finnish or Swedish)	8
Foreign languages · Language A (begun at the lower stage) · Language B (begun at the upper stage)	 8 6
Mathematics	9
Biology, Geography	7
Physics, Chemistry	6
1. Religion / Ethics	3
History, Social studies	6
Music	1
Art	2
Home economics	3
Craft, Technical work, and Textile work	3
Physical education	6
Student Counseling	2
Compulsory subjects, minimum	**70**
Elective subjects, maximum	**20**
Total	**90**

The Council of State Decision on the timetable in the comprehensive school (Helsinki on September 23, 1993).

Appendix 3

1.1. TIME ALLOCATION

1.2. IN THE UPPER SECONDARY SCHOOL

Subject or subject group	1.2.1. Compulsory courses	**Specialisation courses (minimum)**
Mother tongue (Finnish or Swedish)	6	2
Languages		
· Language A (starting from lower comprehensive school)	6	2
· Language B (starting from upper comprehensive school)	5	2
· Other languages	-	16
Mathematics		
· Short level	6	2
· Advanced level	10	3
Natural sciences		
· Biology	2	2
· Geography	2	2
· Physics	1	7
· Chemistry	1	3
Subjects related to human values and beliefs		
· Religion / Ethics	3	2
· Philosophy	1	2

Psychology	-	5
History, Social studies	5	3
Aesthetic subjects · Music · Arts	 1 – 2 1 – 2	 3 3
Physical education, Health education	 3	 3
Careers education and guidance	1	-
Compulsory courses	**45 – 49**	
Specialisation courses minimum	**10**	
Applied courses	**no minimum**	
Total minimum	**75**	

The Council of State Decision on the timetable in the upper secondary school (Helsinki on September 23, 1993).